

HW5 , Math 531, Spring 2014

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QUESTION 1. (i) Let I be an ideal of a commutative ring R. Then the radical ideal of I is denoted by $\sqrt{I}$ where $\sqrt{I} = \{x \in R | x^n \in I\}$. Show that $\sqrt{I}$ is an ideal of R.

(ii) Let R be a commutative ring. Show that $Nil(R)$ is an ideal of R [Hint: since $\{0\}$ is an ideal of R, note that $\sqrt{0} = Nil(R)$]

(iii) Let R be a commutative ring. Show that $R/Nil(R)$ has no nonzero nilpotent elements (i.e., show that $Nil(R/Nil(R)) = \{Nil(R)\}$.

(iv) Find an example of a ring monomorphism $f : R \to S$ where R, S are rings with $1 \neq 0$, but $f(1_R) \neq 1_S$. (Note 1_R indicates the identity of R and 1_S indicates the identity of S)

(v) Let R be a ring with $1_R \neq 0$ and let S be a nontrivial ring (i.e., $S \neq \{0\}$). Suppose that $f : R \to S$ is ring epimorphism. Show that S is a ring with identity $1_S \neq 0$

(vi) Let R, S be rings with $1 \neq 0$ and $f : R \to S$ be a ring homomorphism. Suppose that for some unit u of R we have $f(u)$ is a unit of S. Prove that $f(1_R) = 1_S$ and $f(u^{-1}) = f(u)^{-1}$.

(vii) Let $f : R \to S$ be a ring homomorphism (R, S are rings) such that $f(r) \neq 0$ for some $r \in R$. If R has an identity $1_R \neq 0$ and S has no nonzero zero divisors, then prove that S is a ring with identity $1_S \neq 0$.

Comment: In view of vii, we can conclude that if f is as in vii where R is an integral domain and S is a commutative ring with no nonzero zero divisors, then S must be an integral domain.

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